

Neumann problems on graphs

We discuss Neumann problems for Laplacians on graphs. More specifically, let b be a graph over a set X and let A be a subset of X . Denote the vertex boundary of A by ∂A and suppose that the set $A \cup \partial A$ is finite. In this setting, we introduce a normal derivative ∂_n on the boundary ∂A . We provide existence and uniqueness results for the solution u of the equation

$$\Delta u = 0 \text{ on } A,$$

$$\partial_n u = \varphi \text{ on } \partial A$$

for a given φ on ∂A . Moreover, we prove a probabilistic representation for u .