C*-Algebras

Winter semester 2016/17 **Prof. Dr. M. Keller**

Sheet 6

(1) Let A be an algebra, $a \in A$ and p be a complex polynomial such that $p(a) = 0$. What can be said about $\sigma(a)$?

(2) Let B be a subset of a commutative normed algebra A. Show that

(a)
$$\{ab \mid a \in A, b \in B\} = \bigcap_{B \subseteq I \subseteq A \text{ ideal}} I.$$

(b)
$$\overline{\{ab \mid a \in A, b \in B\}} = \bigcap_{B \subseteq I \subseteq A \text{ closed ideal}} I.$$

(3) Let $\mathcal{K}_1$ be the unital C^*-algebra associated to the non-unital C^*-algebra $\mathcal{K} = \mathcal{K}(H)$ of compact operators on a infinite-dimensional operators on a Hilbert space H and let I be the identity operator. Show that

$$\mathcal{K}_1 \to \mathrm{lin}\{\mathcal{K}, I\} \subseteq \mathcal{L}(H), \quad (a, \lambda) \mapsto a + \lambda I$$

is an isometric isomorphisms.

(4) Give an example of a (non-commutative) Banach algebra that has

(a) no non-trivial ideals,

(b) no characters.