Tunnels of Positive scalar curvature

Dodziuk

Random walks on ramanujan digraphs & complexes

Ori Parzanchevski

k-reg

$$\mathrm{spec}(A) \subseteq \{-k\} \cup [-2\sqrt{k-1}, 2\sqrt{k-1}] \cup \{k\}$$

Marcus - Spielman - Srivastava '13 (bipartite)

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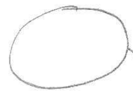
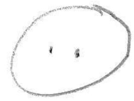
3 candidates

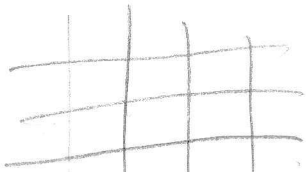
$\mathrm{Cay}(F_k)$  $\sqrt{k}$

$\mathrm{Cay}(FSG_k)$  $\sqrt{k}$

$LDG(T_{k+1})$ line-digraph  $\sqrt{k}$

$$\frac{\lambda \pm \sqrt{\lambda^2 - 4k}}{2}$$

On infinite Ramanujan graphs | Tatiana Nagnibeda



X : d-reg

$P = \frac{1}{d} A \qquad L = 1 - P$

$\rho(X)$

amenable

$T \cong T_d \qquad 1 \geq \rho(X) \geq \rho(T) = \frac{2\sqrt{d-1}}{d}$

infinite ram graph $\quad r(X) = \rho(T)$

$r(X) = \max |\lambda| \neq 1$

$\rho(X) = \frac{1}{d} \limsup (W_n)^{1/n} \qquad \mathrm{cogr}(X) = \limsup (C_n)^{1/n} \in [1, d-1]$

Ramanujan $\Leftrightarrow \mathrm{cogr}(X) \leq \sqrt{d-1}$

∂T, $\quad \nu \qquad C \quad D$

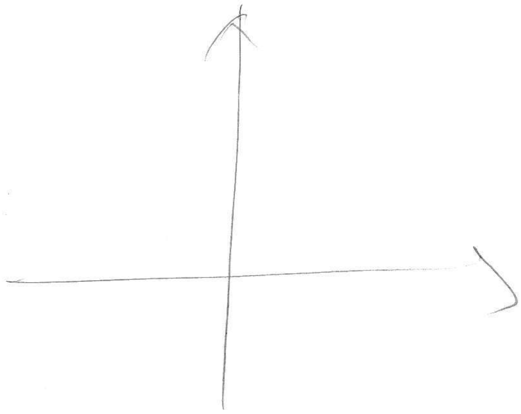
infinitely loops $\quad$ finitely

Topological resonances on q.g. | Francoise True

resonances

poles of $(k^2-\Delta)^{-1}$ $\quad \operatorname{Im} k \leq 0$

$k = \sigma + i\tau$



type I

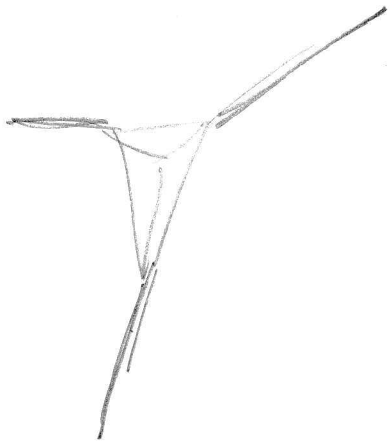
tree with at most one leaf

no vanishing comp' supp

$\exists$ gap

type II

else



$\not\exists$ gap

$\exists \; 0 < h_G < \infty$

$\updownarrow \, h_G/L$

$h_G = 0$

$$\frac{\#\{\text{res in } D_{\varepsilon,K}\}}{K} \xrightarrow[K\to\infty]{} N(\varepsilon)$$

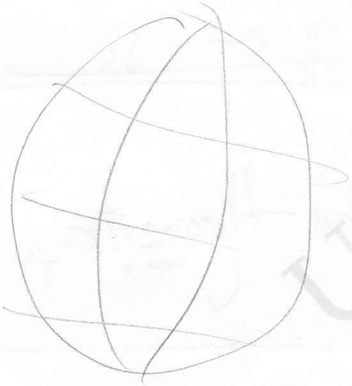
conj

$$N(\varepsilon) \underset{\varepsilon \to 0}{\sim} C \varepsilon^{d(G)/2}$$

$$d(G) = \min\left(g(G) - 1, \# V_0\right)$$

Quotients | Chris

$\pi(r)f = f \cdot\neq -f$ Even



$$L^2(\Gamma/V) \simeq \mathrm{Hom}(V, L^2(\Gamma))$$

$$E_{op_\rho}(\lambda) \cong \mathrm{Hom}(V_\rho, E_{op}(\lambda))$$

Isoperimetric \ Ho

$$i_p := \frac{|\partial S|}{vol(S)}$$

Discrete Hodge Laplacian \ Torki-Hamza

Shnol-type thm for Agmon G.S. | Siegfried Beckus

u gen & poly bd $\Rightarrow E \in \sigma(H)$

$A: \Omega \to \mathbb{R}^d$

$$q(u,v) := \int_\Omega \langle A \nabla u, \nabla v \rangle + V u \bar{v}$$

Weyl sequences $w_n := \frac{u \varphi_n}{\|u \varphi_n\|}$

u generalized & $|u| \leq \varphi \Rightarrow E \in \sigma(H)$

Agmon

$$H^2(\Gamma/R) \cong \mathrm{Hom}_G(V_R, H^2(\Gamma))$$

$$\begin{tikzcd} \mathbb{C}^d \arrow[r, "\theta"] \arrow[d, "Op_{\rho_p}"'] & \varphi \in \mathrm{Hom}_G(V_R, V_\pi) \arrow[d, "Op"] \\ \mathbb{C}^d \arrow[r, "\theta"] & \mathrm{Hom}_G(V_R, V_\pi) \end{tikzcd}$$

K_G

$1 \otimes Op$

K_G

$\forall v \in V_R \quad \varphi(v) \in V_\pi$

$\forall v \in V_R \quad (Op\,\varphi)(v) = Op(\varphi(v))$

$$\begin{array}{cc} & r \\ p & \begin{pmatrix} & \\ & \\ & \end{pmatrix} \end{array}$$

$E^{\alpha_p}_{\sigma_p(\lambda)} = \mathrm{Ker}(\Delta - \lambda 1)$

$v \in \mathbb{C}^d$

$$(1 \otimes Op)\,\theta v \overset{?}{=} \theta\, Op_{\rho} v$$

$\theta^* \theta = 1$

Non - commutative

u harmonic on $\bigcirc$

$\Delta^k(u^2) \geq 0$

$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

Nilpotent groups: $\mathbb{H}_3(\mathbb{Z})$

$N-1$ $\quad$ N

Harmonic fcs - positivity & convexity | Dan Mangoubi

$\binom{N}{k} = \binom{N-1}{k} + \binom{N-1}{k-1}$

$\Delta u = 0$

$q_u(R) = \frac{1}{\mathrm{vol}(S)} \oint u^2$

$q(2R) \leq \sqrt{q(R) \cdot q(4R)}$ 3-circles (Agmon '65)

<u>obs</u> $q^{(k)} \geq 0 \quad \forall k$

<u>Discrete</u>

$\Delta u = 0 \qquad u: \mathbb{Z}^d \to \mathbb{R}$

$Q_u(N) = \mathbb{E}\, u^2(X_N)$

Q_u is also ABS monotonic.

$Q(N) = \mathbb{E}\, u^2(X_N)$

$Q'(N) = \mathbb{E}\, (\Delta u^2)(X_N)$

$\Delta u^2 = 2|\nabla u|^2 \geq 0$

$Q(2N) \leq 2\sqrt{Q(N)\, Q(4N)} + 2^{\sqrt{N}}\, Q(4N)$

sharp for $\binom{N}{k}$

G action on set X

1) $\forall g \in G \quad g: X \to X$

2) "associativity" $\quad g_1 g_2 x = (g_1 g_2) x$

3) $id \circ x = x$

$$g x_1 = g x_2 \Rightarrow (g^{-1} g) x_1 = (g^{-1} g) x_2 \Rightarrow x_1 = x_2$$

$v \in \mathbb{C}^2, \quad v = \begin{pmatrix} a \\ b \end{pmatrix} \quad a, b \in \mathbb{C}$

$g.v$

R

V v.s.

$V_\rho = (\rho, V)$

$W_\pi = (\pi, W)$

G action on V

& lin. action:

$\forall g \in G \quad g(\vec{u} + \vec{v}) = g(\vec{u}) + g(\vec{v})$

$\cancel{\rho(g)}$

$\rho(id) = \rho(g g^{-1}) = \rho(g)\rho(g^{-1})$

$Hom(V, W)$

$\nearrow$

$Hom_G((\rho, V), (\pi, ...)).$

$\boxed{\text{Spectral gaps \& discrete magnetic laplacians} \mid \text{Olaf Post}}$

$$S^{\tilde G} = [0, 2d_\infty] \setminus \sigma(\Delta^{\tilde G})$$

full spectrum conj' $[\text{HS04}]$

& Shirai

abelian cover

$\ast$ mag lap' $\to$ $e^{i\alpha}$ instead of α

G^- delete <u>edges</u>

G^+ virtualise <u>vertices</u> $\times$ $\times$ $\circ$

J_k $\qquad J = \bigcup_k J_k$

$$\bigcup_\alpha \sigma(\Delta_\alpha) \subset J \quad (\ast)$$

$$\lambda(J) = \operatorname{tr} \Delta^{G+} - \operatorname{tr} \Delta^{G-} + 2d_\infty |V_0| \quad (\ast\ast)$$

$(gf)(i) = f(g^{-1}(i))$

$(\pi(g) f)(i) = f(g^{-1}(i))$

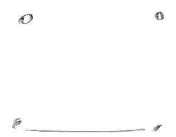
$\pi(g) : V_\pi \to V_\pi$ $\qquad$ $e_i \in V_\pi$

i vertex

V_π^*

$gi = j \iff \pi(g) e_i = e_j$

$= \langle e_i, \pi(g) f \rangle = \langle \pi(g^{-1}) e_i, f \rangle = f(g^{-1} i)$



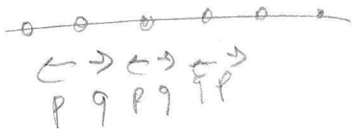
$$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Alan Vallete

Spectral Analysis on singular spaces | Teplyaev

(1)



purely singularly cont'

(2) Schreir graph

gaps in spectral counting func of switched graph

σ

(3) Delone set

Natural Laplacian on hull

Mathematics of aperiodic order
book by Kellendack, Lenz, Savinien

Dirichlet forms on products of fractals | Michael Hinz

$\mathcal{E}_V(f)$

$\Gamma_V(f)(p)$ energy density

δ_0

$\mathcal{E}_n(f) = \left(\frac{5}{3}\right)^n \cdot \mathcal{E}_V(f)$

$\mathcal{E}_K(f) := \lim_{n\to\infty} \mathcal{E}_n(f)$

approx. graphs

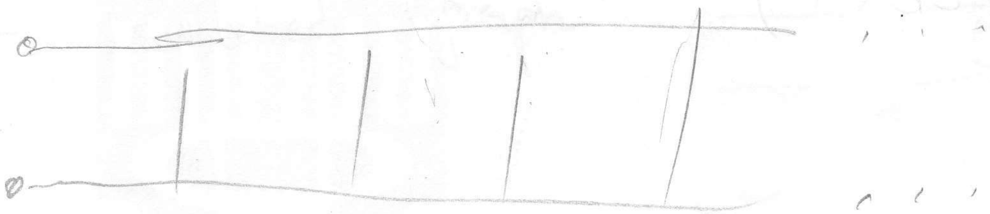
Power dissipation in fractal AC circuits | Patricia Ruiz

ω. φ.

$$\frac{1}{2}|I_{xy}|^2 R(Z_{xy})$$

$$= \frac{1}{2}\frac{R(Z_{xy})}{|Z_{xy}|^2}|V(x)-V(y)|^2$$

x



y

$R(Z^{eff}_{xy}) > 0$ low pass filter

Feynman-Sierpinski ladder

Martin boundary

torus

side length = 2π

level sea.. ✓

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is a Neu dom'

$W^s(x) :=$

$\overline{\quad}$

$\overline{///?}$

Def

Neumann dom'

Neum' line

Some surprises | David Damanik

purely singular continuous

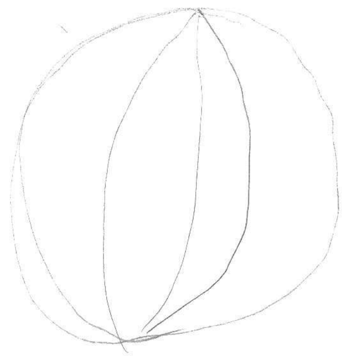
Optimal Hardy type ineq \ Yehuda

Motivation

1)

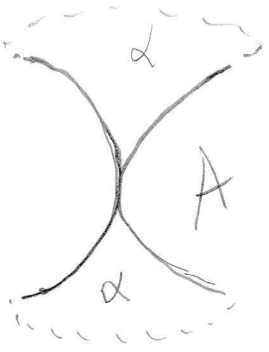
$$|\nabla h|^2 < W$$

2)

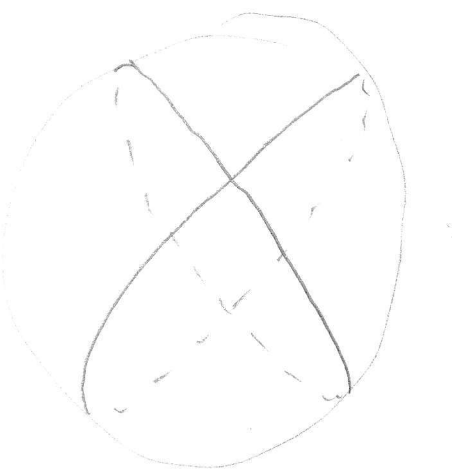


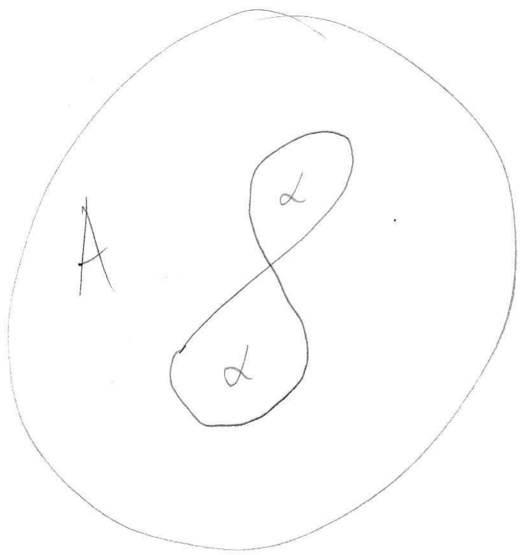
Periodic quantum graphs

Pavel Exner



$A = \pi$





- Mapping to Jitó & Avila
- Exner & Vašata '17

[ ] Cantor set between all evalues

Pavel Exner

A few gaps only

$$(U-\mathbb{1})\psi + i(U+\mathbb{1})\psi' = 0$$

scale-invariant – no Robin

$$\begin{pmatrix} I^{(r)} & T \\ 0 & 0 \end{pmatrix} \psi' = \begin{pmatrix} S & 0 \\ -T^* & I^{(n-r)} \end{pmatrix} \psi$$

Dynamical on hybrid systems	Vsevolod Chernyshev

" Integer points in polyhedra " Barvinok

- Infinite graphs and manifolds.

- Fundamental polygon of the torus

$\boxed{\text{Neumann domains} \mid \text{Sebastian}}$

* $\lambda_2^{(N)} \to \lambda_1^{(N)}$ in the ineq'

$L(q; s_1, \dots s_n) := \langle \gamma \rangle \backslash S^{2n-1}$

$\gamma = \mathrm{diag}(R_{s_1} \quad \dots \quad R_{s_n})$

$L, L' \subset \mathbb{Z}^n$

Strong isospectrality

On the essential spectrum of Sch' Op on trees | Latif

$\sigma_{ess}(H)$ Weyl's criteria

(1) $\lambda \in \sigma_{ess}(H) \iff \exists \{\psi_n\} \in \ell^2(T)$

$\|\psi_n\| = 1$ $\quad \|(H - \lambda_n)\psi_n\| \xrightarrow{n\to\infty} 0$ $\quad (*)$

(2) $\lambda \in \sigma_{ess}(H) \iff$ $(*)$ & $\psi_n \xrightarrow{w} 0$

Right Limits

$J : \ell^2(\mathbb{N}) \circlearrowleft$ $\quad J = \Delta + Q$ bdd

$J^{(r)} : \ell^2(\mathbb{Z}) \circlearrowleft$ is right lim

If

Thm (Last-Simon)

$$\sigma_{ess}(J) = \bigcup_{\text{right limit}} \sigma(J^{(r)})$$