Seminar Geometrie
Lie groups and homogeneous spaces

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Lie groups are groups which are at the same time manifolds in such a way that all algebraic operations are smooth. Thus they are objects at the border of algebra and geometry. They are used to describe symmetries in many different contexts, from analysis to physics.

The seminar will offer an introduction to Lie groups. We will discuss the relation to Lie algebras and some basic theory. Then we will study homogeneous spaces. This class of manifolds is rich enough to contain many different interesting examples but special enough to allow for explicit computation of geometric quantities such as curvature.

Prerequisites. You need to have attended an introduction to differential geometry and be familiar with its basics concepts such as manifolds, isometries and curvature.

In the first session on Thursday, October 19, the program will be explained in more detail and the talks will be assigned.

Program

1. Lie groups and their Lie algebras [3, 3.1–3.12]

2. Homomorphisms [3, 3.13–3.16]

3. Lie subgroups [3, 3.17–3.21]

4. Coverings and simply-connected Lie groups [3, 3.22–3.28]

5. Exponential map [3, 3.29–3.37]

6. Continuous homomorphisms and closed subgroups [3, 3.38–3.43]

7. Adjoint representation and bilinear operations [3, 3.44–3.57]

Literature

[1] Arthur L. Besse, *Einstein manifolds. Reprint of the 1987 edition*, Reprint of the 1987 edition, Berlin: Springer, 2008 (English).

[2] S. Kobayashi and K. Nomizu, *Foundations of differential geometry. Vol. I*, New York-London: Interscience Publishers, 1963 (English).

[3] Frank W. Warner, *Foundations of differentiable manifolds and Lie groups. Reprint*, New York: Springer, 1983 (English).